Santos®: A Unique Physics-Based Virtual Human Model Enabling Scientific Analysis and Prediction of Warfighter Behavior

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ABSTRACT

Modeling and simulation requires higher-fidelity, physics-based human models to conduct proper analysis of human behavior, and to predict how humans will conduct various tasks. The widespread use of "scripted" avatars within modeling and simulation today limits analysis and prediction of human behavior. Thus, organizations have not realized the full cost savings, improved decision aids, and better training promised by modeling and simulation.

The following paper will demonstrate two innovations: first, a description of the most realistic human virtual models available today, known as Santos and Sophia. These models differ from other "scripted" human avatars in that they've been developed using physics-based, high-fidelity predictive analytics, not just motion capture. Santos and Sophia are physiologically accurate (i.e., musculoskeletal system, vascular system), physics-based virtual human models that allow for customized inputs (individual height, weight, stride, strength, fitness, environment, task, etc.) and enable the output of measurable and quantifiable mechanical and physiological human behavior metrics (e.g., fatigue, discomfort, joint torque, hydration and calorie expenditure, VO2).

In other words, Santos and Sophia predict how various humans will complete tasks and then output the human performance and physiology metrics required for analysis. The U.S. Department of Defense (DOD) has used Santos and Sophia to analyze body armor/personal protective equipment for both males and females, to study extended load carriage (analyzing the impact of heavy loads on knees and backs), and now for the Army Combat Fitness Test.

Secondly, we will share a roadmap for the adoption of improved human behavior representation definitions that can inform standards, requirements, and measures of effectiveness and performance. The presentation will contain videos of the TRL9 software, examples of analysis and prediction, and how the technology can help industry.

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INTRODUCTION

There have been many advances in the field of digital human modeling (DHM) and simulation. Many tools are now available for modeling digital humans within virtual environments to facilitate designing products, evaluating capabilities, training workers and warfighters, predicting injuries, and simulating real-world scenarios. As the use of modeling and simulation expands, the demand for better human models has increased substantially across a variety of industries to evaluate the effects of changes in design space on all human aspects. Human behavior changes in response to changes in the objects and environment that define the scenario. For instance, the walking gait changes when the terrain changes from concrete ground to sand or ice, or the loading conditions change from no load to carrying a backpack, or the environment changes from flat ground to ground with obstacles. There is a growing need to analyze and evaluate human performance under different design criterions, loading conditions, or environmental conditions. However, most current methods of motion simulation lack the adaptability to account for changes in simulation parameters, including body types, workspace configurations, and equipment designs, and can produce unrealistic simulations. Human models currently used for modeling and simulation applications lack the fidelity and realism necessary to show how individuals' load carriage, environment (heat, humidity, altitude), and individual differences (fitness, height, weight) affect how they perform various tasks. This lack of realism has and will continue to produce negative experiences for decision makers using these tools. To realistically simulate these scenarios, we need physics-based models that account for human limitations, external loads, and environmental factors.

The Santos® human simulation environment, shown in Figure 1, was developed from the ground up starting in 2003 at the University of Iowa's Virtual Soldier Research (VSR) program. Santos® uses an optimization-based approach called human predictive dynamics to simulate human motion.

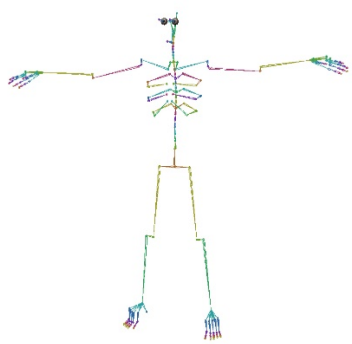
Figure 1. Scenario Generation and Simulation Capabilities within the Santos Environment with Multiple Physics-based Avatars

This approach allows for predictive capabilities based on changes in internal parameters, such as anthropometry and body type, and external parameters, such as load carriage and obstacles in the environment, while accounting for physics-based limitations such as strength and fatigue to be incorporated in the model. Most unique to the Santos model is its ability to conduct dynamic tasks, showing the effect of various simulation parameters on the performance of the task. A user of the simulation environment can define a task, design a human avatar, and execute the motion. The result is Santos attempting the task and reporting back to the user whether or not the task is feasible and reporting detailed analytical feedback if the task is feasible (Abdel-Malek et al., 2008; Abdel-Malek et al., 2013; Kim et al., 2009; Xiang et al., 2010; Marler et al., 2012). Santos has also been used in many military applications, from designing vehicles, weapons, and systems to studying extended load carriage capabilities for the U.S. Marines (Bhatt et al., 2008; Chung et al., 2015; Degenhardt et al., 2014; Hariri et al., 2013; Johnson et al., 2010; Kim et al., 2009; Knake et al., 2010; Kwon et al., 2014). The model also includes a comprehensive physiology system that includes strength, fatigue, and cardiovascular parameters, as well as a complete musculoskeletal model.

DIGITAL HUMAN MODEL

Human simulation offers decision makers the chance to experiment with different anatomies and movements without having to experiment with actual human beings. Human simulation includes accessing a computer model of a human, also known as an avatar or a digital human model (DHM), and generating motion sequences using these models. A good DHM is not only a detailed avatar but is also parametric, allowing for variability in kinesiology such as joint angle profiles and ground reaction forces, as well as input parameters such as step length and external load, and can react to changes in the environment and task parameters. Below is a brief description of Santos, the DHM developed at the University of Iowa.

Visual DHM



(a) Skeletal structure

(b) Male skin on skeleton – Santos avatar

(c) Female skin on skeleton – Sophia avatar

Figure 2. Skeletal and Skinned Visual Models of Santos and Sophia

Traditional 3D modeling, texturing, and animation rigging techniques are used to ensure visual realism of the Santos avatar. The visual 3D model includes the “skin” of the avatar. This “skin” is comparable to an infinitely thin but hollow shell that defines the avatar’s shape. This skin allows visual representation of avatars of different gender, body type, and anthropometric parameters. One way to create this visual representation of skin is by use of 3D scanning techniques on real humans. The skin can then be altered by an artist to make the avatar have different body types. Once the shape of the avatar is created, shaders (a compilation of effects that dictate how a 3D surface responds to light) and textures (2D images that are projected onto or wrapped around 3D surfaces) are used to provide the avatar’s shape with the visual cues necessary to create the illusion of human skin

While such a 3D model is a realistic-looking virtual image of a human, it cannot be used to generate simulations that involve motions of various joints of the model. A hierarchical structure, referred to as an inverse kinematics (IK) skeleton, is then added to the model, which includes a series of interdependent local coordinate systems positioned strategically to enable the creation of joints such as knees, elbows, shoulders, etc. Figure 2 shows the skeleton in the context of the complete model. The avatar's skin is then bound to the IK skeleton, allowing the avatar to move just as a human does. These joints, in combination with skin weighting techniques, allows the movement of the limbs of the model in an aesthetically pleasing manner.

Physics-based DHM

The versatility and effectiveness of methods for simulating human motion are largely dependent on the underlying mathematical models used to represent the human skeletal system. Santos models the human skeleton as a series of rigid links connected by kinematic joints to represent the human skeletal system. The DHM is a three-dimensional, 215-degree-of-freedom, rigid-link, articulated mechanical structure based on the Denavit-Hartenberg (DH) parameterization (Denavit & Hartenberg, 1955). The DH parameterization is a matrix transformation method to systematically describe the translational and rotational relationship between adjacent reference frames in an articulated chain. Kinematically, the digital human is a branched structure with seven branches. The first branch contains the six global degrees of freedom (DOFs) (three translational and three rotational) that reference the location of the hip point on the digital human with respect to an inertial reference frame located on the ground. The other six branches correspond to the right leg, left leg, spine, right hand, left hand, and head.

Figure 3. Use of Parametric Digital Human Model to Easily Generate Many Avatars with Differing Anthropometries

One benefit of the DH method is that it can efficiently represent the transformation from one DOF frame to the next in the kinematic chain using four parameters (Farrell, 2005) instead of the usual six parameters. This reduced parameterization is achieved by imposing certain restrictions on how the successive 3D frames are arranged. Another benefit of DH parameterization is its ability to be implemented iteratively. Thus, for large and computationally intensive systems such as humans, it is relatively easy to develop a model that is suitable for computer implementation. In addition, changing the parameters of link lengths and mass distribution allows the designers to create a number of human avatars, as shown in Figure 3, to test products.

One of the restrictions imposed on successive 3D frames is that the DH method only allows a single DOF between any two consecutive frames. However, humans have joints with multiple DOFs, such as spine, wrist, and hip joints. The frames at these joints are modeled as collocated DH frames to adhere to the DH notation. Hence, for any joint in the human body that has more than one DOF, one or more virtual links with zero length are inserted between two consecutive joints. For instance, one virtual link is inserted between the two joints in the ankle, and two virtual links are inserted between the three collocated joints in the spine, as shown in Figure 4. The mass and inertia properties of each body segment are estimated based on the Air Force Research Lab's Articulated Total Body software for any avatar with given height and weight (Cheng et al., 1996).

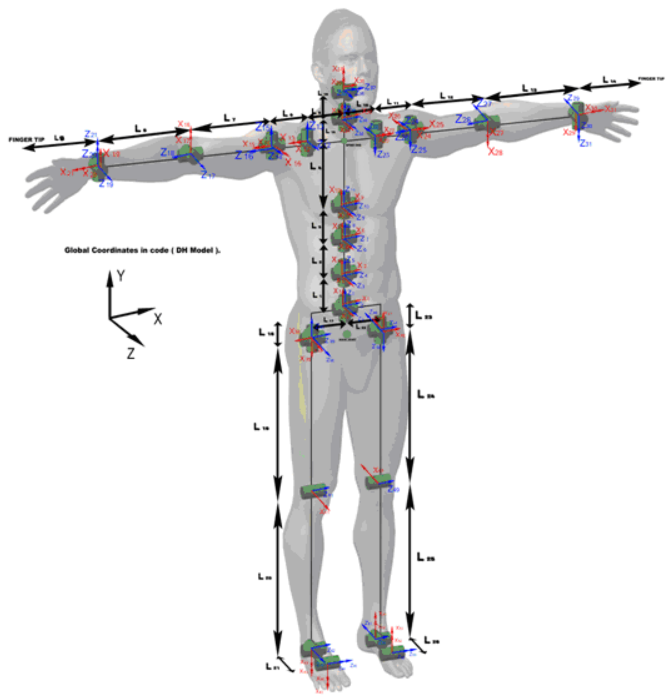


Figure 4. The Underlying DH-based Mathematical Structure of Santos that Enables Various Link Articulations and Provides a Foundation for the Computational Model

Recursive Kinematics and Dynamics

One of the ways in which physics is introduced in the DHM is by evaluating the kinematics and dynamics and making sure that all physics-based constraints arising from forces and torques are valid throughout the simulation. Santos uses recursive Lagrangian kinematics and dynamics for an efficient calculation of kinematics and dynamics, which is vital to generating physically accurate digital human simulations. This standard recursive approach is summarized below.

Let the transformation matrix $\mathbf{T}_i$ be the ith link transformation that relates the ith and i-1th local reference frames. This matrix $\mathbf{T}_i$ can be expressed as

$$\mathbf{T}_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The recursive kinematics calculations of this transformation matrix lead to a cumulative transformation matrix $\mathbf{A}_i$:

$$\mathbf{A}_i = \mathbf{T}_1\mathbf{T}_2\mathbf{T}_3 \ldots \mathbf{T}_{i-1}\mathbf{T}_i = \mathbf{A}_{i-1}\mathbf{T}_i \quad (2)$$

Thus, the kinematic position of a point of interest in the ith frame, ${}^{i}\mathbf{x}$, can be expressed in the global reference frame, ${}^{0}\mathbf{x}$, using the cumulative transformation matrix as:

$${}^{0}\mathbf{x} = \mathbf{A}_i \, {}^{i}\mathbf{x} \quad (3)$$

The above equation can be used to calculate the global position of a point on the body for any given joint angle values. The DH parameters θ_i, d_i, α_i, and a_i that define the transformation matrix $\mathbf{T}_i$ are defined in a very specific manner. For each frame, the i^{th} z-axis controls the motion for the $(i + 1)^{th}$ DOF. Farrell (2005) describes the four parameters of the method that are used to obtain the position and orientation of frame i with respect to frame $i - 1$, as follows:

Angle θ_i between the $(i\text{-}1)^{th}$ and i^{th} x-axis about the $(i\text{-}1)^{th}$ z-axis

Distance d_i from the $(i\text{-}1)^{th}$ to the i^{th} x-axis along the $(i\text{-}1)^{th}$ z-axis

Angle α_i between the $(i\text{-}1)^{th}$ and i^{th} z-axis about the i^{th} x-axis

Distance a_i from the $(i\text{-}1)^{th}$ to the i^{th} x-axis along the i^{th} x-axis

Time derivatives of the transformation matrix $\mathbf{A}_i$ are necessary to calculate further kinematic quantities, such as velocities and accelerations, as well as dynamic quantities, such as the torques, and can also be obtained in the recursive form as:

$$\mathbf{B}_i = \dot{\mathbf{A}}_i = \mathbf{B}_{i-1}\mathbf{T}_i + \mathbf{A}_{i-1}\frac{\partial \mathbf{T}_i}{\partial q_i}\dot{q}_i \tag{4}$$

$$\mathbf{C}_i = \dot{\mathbf{B}}_i = \mathbf{C}_{i-1}\mathbf{T}_i + 2\mathbf{B}_{i-1}\frac{\partial \mathbf{T}_i}{\partial q_i}\dot{q}_i + \mathbf{A}_{i-1}\frac{\partial^2 \mathbf{T}_i}{\partial q_i^2}\dot{q}_i^2 + \mathbf{A}_{i-1}\frac{\partial \mathbf{T}_i}{\partial q_i}\ddot{q}_i \tag{5}$$

where q_i is the generalized coordinate for transformation $\mathbf{T}_i$, $\mathbf{A}_0 = \mathbf{I}$ and $\mathbf{B}_0 = \mathbf{C}_0 = \mathbf{0}$.

The Lagrange's equation, which evaluates the dynamics of the system using the recursive formulation, is given as:

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} \tag{6}$$

where $L = K - V$ (kinetic energy – potential energy), q is the generalized coordinate vector (joint angles), and τ_i is the joint torque vector. Given the mass and inertia properties of each body segment and the external force $\mathbf{f}_k^T = \begin{bmatrix} f_x & f_y & f_z & 0 \end{bmatrix}$ and moment $\mathbf{h}_k^T = \begin{bmatrix} h_x & h_y & h_z & 0 \end{bmatrix}$ for the link k $(1 \le k \le n)$ defined in the global coordinate system, the joint actuation torques τ_i for each of the joints can be computed using recursive backward dynamics as follows:

$$\tau_i = tr\left[\frac{\partial \mathbf{A}_i}{\partial q_i}\mathbf{D}_i\right] - \mathbf{g}^T\frac{\partial \mathbf{A}_i}{\partial q_i}\mathbf{E}_i - \mathbf{f}_k^T\frac{\partial \mathbf{A}_i}{\partial q_i}\mathbf{F}_i - \mathbf{G}_i^T\mathbf{A}_{i-1}\mathbf{z}_0 \tag{7}$$

where

$$\mathbf{D}_i = J_i\mathbf{C}_i^T + \mathbf{T}_{i+1}\mathbf{D}_{i+1} \tag{7a}$$

$$\mathbf{E}_i = m_i\,{}^i r_i + \mathbf{T}_{i+1}\mathbf{E}_{i+1} \tag{7b}$$

$$\mathbf{F}_i = {}^k\mathbf{r}_f\,\delta_{ik} + \mathbf{T}_{i+1}\mathbf{F}_{i+1} \tag{7c}$$

$$\mathbf{G}_i = \mathbf{h}_k \delta_{ik} + \mathbf{G}_{i+1} \tag{7d}$$

$$\mathbf{D}_{n+1} = \mathbf{E}_{n+1} = \mathbf{F}_{n+1} = \mathbf{0} \tag{7e}$$

where $\mathbf{g}$ is the gravity vector, ${}^{i}\mathbf{r}_i$ is the location of the center of mass in the i^{th} local frame, ${}^{k}\mathbf{r}_f$ is the location of the external force acting in the k^{th} frame, and δ_{ik} is the Kronecker delta. The gradients of the equations of motion (EOMs) are also required for faster implementation of gradient-based optimization methods. These gradients with respect to joint angles, joint angle velocities, and joint angle accelerations can also be analytically calculated (Xiang et al., 2009).

OPTIMIZATION

The EOMs must be solved to determine the forces and torques required to perform any given task. Many different approaches are available in the literature, each with its own advantages and disadvantages (Otten, 2003). However, the traditional approaches to solving EOMs work well with open-chain systems with relatively fewer DOFs. Human posture and motion simulation involves solving EOMs of a much larger degree system and is a very time-consuming and computationally intensive task. The complexity and challenge increase substantially, since most human tasks would require the model to alternate between closed-loop and open-chain systems. Traditional EOMs solvers are also not well suited for applications where there are intermediate constraints on the state of the variables. For instance, to simulate the walking task, the initial and final foot placement is typically constrained to be on the floor, thus making it a closed-loop system. In addition, there might be intermediate constraints on the hands to simulate an activity being performed by the human while walking, such as carrying a box. Hence, we employ an optimization-based strategy to estimate the solution of a large, inter-dependent system of EOMs. This approach also allows us to expand the modeling capabilities by adding more components to the objective function and constraints as the research matures.

A typical optimization problem has three main components: design variables, an objective function, and constraints (Arora, 2016). Design variables are the parameters of the problem that can change and the values of which need to be determined. The objective function, a function of design variables, is a measure that must be minimized. For the problem of human modeling and simulation, we use one or many performance measures as the objective function. The constraints are the bounds on the design variables or the bounds on some function of the design variables that must not be exceeded. Below, we describe these components for a typical human posture and motion simulation problem (Bhatt et al., 2019).

Design Variables

Design variables are those parameters of the problem that need to be determined. For a human posture determination problem, joint angle values, q_i, are the design variables. When the problem involves a time component, as in motion simulation, joint angle profiles $q_i(t)$ are the design variables. However, as a function of time, any joint angle profile has infinite values that need to be determined. Thus, design variables are approximated as linear combinations of cubic B-spline basis functions. In this case, the control points of these cubic B-spline functions serve as the finite number of design variables. Corresponding joint angle, velocity, and acceleration values are calculated at each iteration from these control point values.

Performance Measure

The goal of the optimization process is to minimize/maximize some performance criteria. In the case of human modeling and simulation, the performance criteria typically represent real-world driving factors for human posture and motion. For example, human posture can be driven in part to minimize discomfort while assuming a pose to do or touch something, to acquire a visual target, or to achieve both (minimizing discomfort while acquiring a visual target). In optimization-based human simulation, these performance criteria are modeled as mathematical functions of the design variables that are minimized to guide the solution. Some common performance measures are joint displacement, joint discomfort, vision, and joint torque.

Constraints

Several physics-based, task-based, and environment-based constraints must be employed to predict the postures and motions. A DHM can behave as a superhuman whose strength has no limits and who does not get tired in the absence of any constraints. The physics-based constraints typically limit the capabilities of the human model to avoid unrealistic or impossible situations like lifting a heavy table with one finger. Joint angle limits, joint torque limits, self-avoidance, and stability criteria are examples of these physics-based constraints. Task-based constraints are dependent on the task being simulated. For instance, if the DHM is used to simulate the task of applying force to the handbrake of a vehicle, it is necessary for the hand to touch the handbrake in a specific location at a particular angle. Hence, the position and angle constraints for the hand will be task-based constraints added to the problem. The environment-based constraints depend on the task as well as the environment surrounding the human in which the task must be simulated. For instance, the feet must never go below the floor, and body segments cannot go into obstacles or walls. A few common constraints for the predictive algorithms use include distance, vision, self-avoidance, collision avoidance, and strength limits. Constraints allow flexibility in optimization-based human simulation. Additional constraints have been and can be modeled to expand the current capabilities.

Figure 5. An Intuitive Graphical User Interface Developed with Drag-and-Drop Capabilities as well as Intuitive Visualization Tools to Present Complex and Large Amounts of Analytical Data

The optimization-based approach used in Santos allows the addition of capabilities by developing new performance measures or by augmenting existing ones with additional criteria. In addition, the constraints can be added when modeling any new factor that limits or enhances human motion. For instance, a collision-avoidance constraint can be added if there are obstacles in environment.

STRENGTH

The strength limits of a human for any given joint change dynamically as the position of the joint changes and as the speed and direction of the joint movement change. Most DHMs either do not include strength as a limitation or include only peak strength as a limiting factor. Santos, being optimization-based, models such dynamic strength as a constraint for all major joints (Marler, et al., 2012). This implementation allows Santos to simulate situations and answer the question of which percentile population is able to perform that task. When implemented as a constraint, it also answers whether the task can be performed by a human or any percentile. If the task cannot be performed as is by a human with a given strength, Santos can provide alternative ways, if possible, to perform it.

PHYSIOLOGY

The mathematical formulation of Santos also allows for prediction of metabolic energy consumption as any dynamic task is being performed by Santos (Bhatt, et al., 2016). The estimation of metabolic energy is based on the mechanical energy requirements for the task while also accounting for basal energy as well as heat loss. Santos can also estimate other physiological parameters such as oxygen consumption, hydration, calorie expenditure, and heart rate.

GRAPHICAL USER INTERFACE

A fully parametric DHM and simulation software that also models physics-based constraints can be very complex. A user is faced with a steep learning curve and must have knowledge of many different fields, including human anatomy and physiology. The intended user may not have such a wide variety of knowledge. Hence, VSR also develops custom interfaces for the Santos software depending on the need of individuals. These software are geared to help a well-defined end user. Once such software is GruntSim, which was designed for use by the Marines, as shown in Figure 5. It allows the user to select an avatar from 14 specific boundary cases, load that avatar with provided Marine equipment, and perform various analyses, such as range of motion, agility, strength, vision, stability, task performance, and so on. It also allows the user to perform trade-off analysis to reduce schedule and field trials before fielding new equipment.

A DHM developed with such a ground-up approach differs from other “scripted” human avatars since it has been developed using physics-based, high-fidelity predictive algorithms, not just using and reproducing motions from motion capture. Santos DHMs are physiologically accurate (i.e., musculoskeletal system, vascular system), physics-based virtual human models that allow for customized inputs (individual height, weight, stride, strength, fitness, environment, task, etc.) and enable the output of measurable and quantifiable mechanical and physiological human behavior metrics (e.g., fatigue, discomfort, joint torque, hydration and calorie expenditure, VO2).

CONCLUSION

This paper describes Santos and Sophia digital human models developed by the VSR Lab at the University of Iowa. These advanced physics-based DHMs differ substantially from contemporary “scripted” human avatars developed primarily using motion capture technology. Use of optimization-based predictive dynamics allows Santos and Sophia to be physiologically accurate (i.e., musculoskeletal system, vascular system) virtual human models that allow for customized inputs (individual height, weight, stride, strength, fitness, environment, task, etc.) and enable the output of measurable and quantifiable mechanical and physiological human behavior metrics (e.g., fatigue, discomfort, joint torque, hydration and calorie expenditure, VO2).

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